

--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

VASAVI COLLEGE OF ENGINEERING (*Autonomous*), HYDERABAD
B.E. I Year I-Semester (Old) Examinations, May/June-2016

Mathematics-I

Time: 3 hours

Max. Marks: 70

Note: Answer ALL questions in Part-A and any FIVE from Part-B

Part-A (10 × 2 = 20 Marks)

- 1) Find the interval in which 'b' should lie so that both the Eigen values of the matrix $\begin{bmatrix} -1 & 3 \\ b & -2 \end{bmatrix}$ are negative.
- 2) What is the sum of Eigen values of a matrix equal to ____.
- 3) In a series of Positive terms, if the nth term tends to a finite value as n tends to infinity, write the comment on the convergence of the series.
- 4) Examine the convergence of the series: (i) $\sum \frac{n}{1+2^{-n}}$
- 5) Define Evolute of a curve. Write the curve for which the radius of curvature at any point on it is constant.
- 6) Write any 4 properties of a curve generally used for tracing of a curve.
- 7) Find the Jacobian of Cartesian coordinates with respect to polar coordinates.
- 8) If $z = f(x, y, p)$ and $x = g(t), y = h(t), p = k(t)$, then find the total derivative of z with respect to t.
- 9) In a double integral of a function of two variables x and y, both the limits of the variables are constants, then what is the region of integration in the XY plane.
- 10) Compute $\int_{-1}^4 \int_{x^2-10}^{3x-6} x^2 y dy dx$

Part-B (Marks: 5 X 10= 50)
(All bits carry equal marks)

- 11) (a) Find the Eigen values and Eigen Vectors of $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$
- (b) Reduce the quadratic form $3x_1^2 + 3x_2^2 + 3x_3^2 + 2x_1x_2 + 2x_1x_3 - 2x_2x_3$ to canonical form by orthogonal transformation. Find rank, index and signature of the quadratic form.

12) (a) (i) Test whether the following series is Absolutely / Conditionally convergent

$$\sum_{n=1}^{\infty} (-1)^{n+1} (\sqrt{n+1} - \sqrt{n})$$

ii) Examine the convergence or divergence of $\frac{2 \cdot 1}{3^2 \cdot 4^2} + \frac{4 \cdot 3}{5^2 \cdot 6^2} + \frac{6 \cdot 5}{7^2 \cdot 8^2} + \dots$

(b) Define (i) alternating series (ii) Absolute convergence of a series (iii) Conditional convergence of a series. Also test the Absolute convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1}$

13) (a) Trace the curve $y^2 = (x-2)(x-4)^2$.

(b) Find the Evolutes of the parabola $y^2 = 4ax$.

14) (a) Find the point on the sphere $x^2 + y^2 + z^2 = 1$, nearest to the point (2, 1, 1) using method of Lagrange's multipliers.

(b) If $u(x, y) = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right)$, $x > 0, y > 0$, then find $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$.

15) (a) Evaluate $\int_0^{\log 2} \int_0^x \int_0^{x+y} e^{x+y+z} dx dy dz$

(b) Evaluate $\int_0^{\infty} \int_x^{\infty} \frac{e^{-y}}{y} dy dx$ by changing the order of integration

16) (a) Using Cayley-Hamilton Theorem and find A^{-1} , where $A = \begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix}$

(b) Test the convergence of the following series (i) $\sum \frac{1}{\sqrt{n} + \sqrt{n+1}}$ (ii) $\sum_{n=1}^{\infty} \frac{x^{n-1}}{n \cdot 3^n}$ ($x > 0$)

17) (a) Find the envelop of the family of straight line $\frac{x}{a} + \frac{y}{b} = 1$ where a, b are connected by the relation $a + b = c$

(b) Find the maximum / minimum values of $f = x^2 y^2 - 5x^2 - 8xy - 5y^2$
